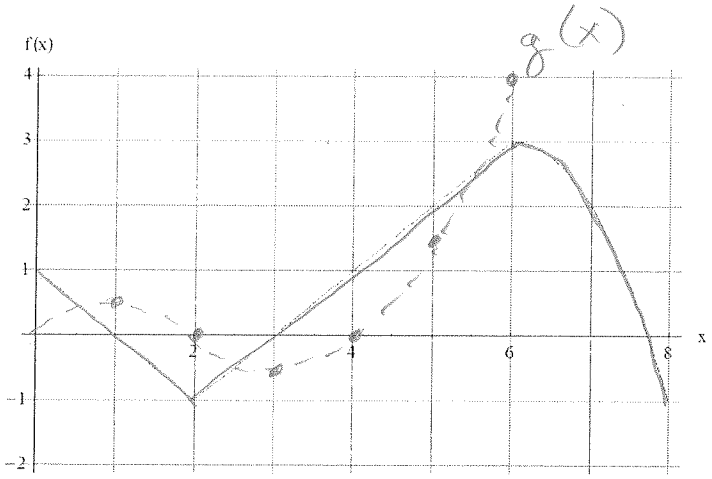
Mechanics Based Problems

1. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown below.
   (a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.
   (b) Estimate $g(7)$.
   (c) Where does g have a maximum value? Where does it have a minimum value?
   (d) Sketch a rough graph of g.



a) $g(0) = 0$

$g(1) = \frac{1}{2}$

$g(2) = 0$

$g(3) = -\frac{1}{2}$

$g(4) = 0$

$g(5) = \frac{3}{2}$

$g(6) = 4$

b) $g(7) \approx 6.7$

c) MAX @ $x = 7$

MIN @ $x = 3$

2. Let $g(x) = \int_{-3}^{x} f(t)dt$, where f is the function whose graph is shown below.

   (a) Evaluate $g(3)$ and $g(-3)$.

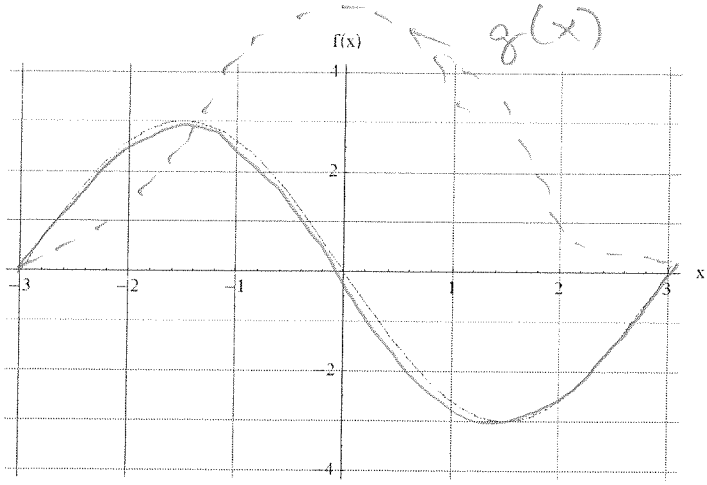
   (b) Estimate $g(-2)$, $g(-1)$, and $g(0)$.

   (c) On what interval is g increasing?

   (d) Where does g have a maximum value?

   (e) Sketch a rough graph of g.

   (f) Use the graph in part (e) to sketch the graph of $g'(x)$. Compare it with the graph of f



a) $g(3) = 0$

b) $g(-2)$, using midpt $= 1.5$.

$g(-1) = 4.5$

$g(0) = 6$

c) $-3 \leq x \leq 0$

d) MAX @ $x = 0$

e) sketch above

f) $g'(x)$ is $f(x)$

3. Use part I of the Fundamental Theorem of Calculus to find the derivative of the function:

$$g(x) = \int_1^x \ln(t)dt$$

$g'(x) = \ln(x)$

Problem Solving Problems

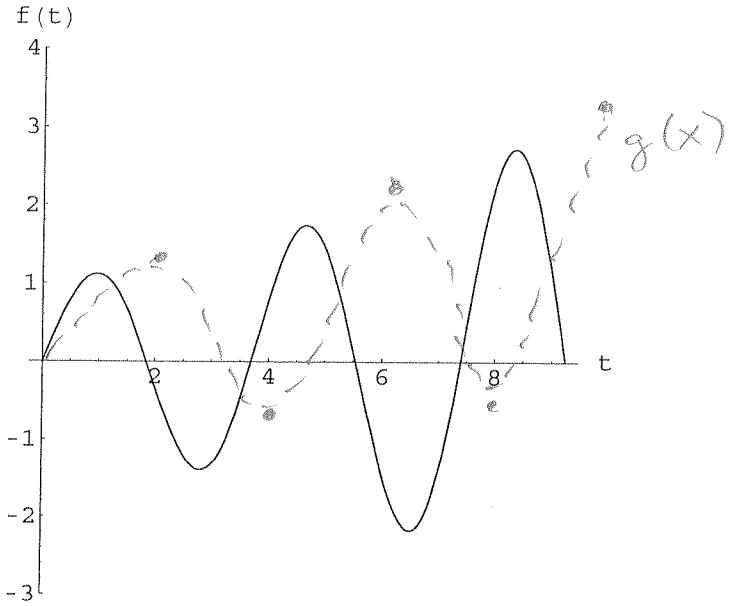
1. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown below.

   (a) At what values of x do the local maximum and minimum values of g occur?

   (b) Where does g attain its absolute maximum value?

   (c) On what intervals is g concave downward?

   (d) Sketch the graph of g.



a) local max @ $x \approx 1.9, 5.5, 9.2$

local min @ $x \approx 0, 3.6, 7.4$

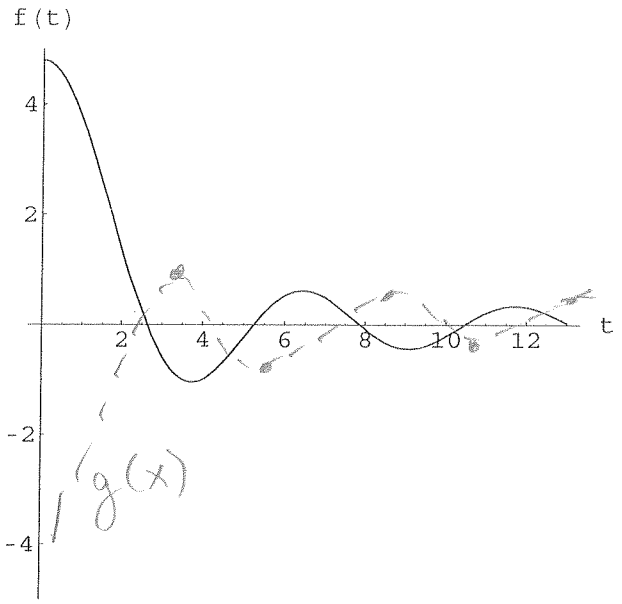
b) Abs max @ $x \approx 9.2$

c) concave down: $1 \le x \le 2.9$

$4.6 \le x \le 6.5$

$8.5 \le x \le 9.5$

2. Let $g(x) = \int_2^x f(t)dt$, where f is the function whose graph is shown below.



(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

(c) On what intervals is g concave downward?

(d) Sketch the graph of g.

a) local max @ $x \approx 2.6, 7.9, 13$

local min @ $x \approx 2, 5.5, 10.5$

b) abs MAX @ $x \approx 2.6$

c) concave down: $2 \leq x \leq 3.5$

$6.5 \leq x \leq 9$

$11.6 \leq x \leq 13$

3. Use part I of the Fundamental Theorem of Calculus to find the derivative of the function:

$$F(x) = \int_x^{10} \tan(\theta) d\theta$$

Hint:

$$\int_x^{10} \tan(\theta) d\theta = -\int_{10}^{x} \tan(\theta) d\theta$$

$f(x) = F'(x) = -\tan(x)$