Lesson 47 - Spring Mass Systems II

Objectives

- Model initial value problems involving a spring mass system.
- Solve problems involving, free undamped, damped, and driven motion.
- Understand critically damped, under damped, and over damped spring mass systems.
- Be able to describe the long term behavior of a spring mass system.

READ

- Review the previous lesson's reading.

THINK ABOUT

- What is the form for any linear second order differential equation?
- What is meant by constant coefficients?
- What exactly does damping do?
- What exactly does a spring constant do to a spring?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

No new commands.

Problem Solving Problems

1. A spring with a 4 kg mass has natural length 1 m and is maintained stretched to a length of 1.3 m by a force of 24.3 N. If the spring is compressed to a length of 0.8 m and then released from rest,

   (a) Find the position of the mass at any time t.

   $x(t) = -0.5\cos(\frac{9}{2}t)$

   (b) Find the damping coefficient that would produce critical damping.

   $C = 36$

2. A 16 lb weight stretches a spring 4 ft. The spring-mass system is in a medium with a damping constant of 4.5 lb-sec/ft.

   (a) What is the solution function describing the position of the mass at any time if the mass is released from 2 ft below the equilibrium position with an initial velocity of 4 ft/sec downward?

   $x(t) = \frac{20}{7}e^{-t} - \frac{6}{7}e^{-8t}$

   (b) Find the minimal spring constant that would produce underdamped behavior.

   $k > \frac{81}{8}$

3. A 16-lb weight is attached to the lower end of a coil spring suspended from the ceiling and having a spring constant of 1 lb/ft. The resistance in the spring-mass system is numerically equal to the instantaneous velocity. At $t = 0$ the weight is set in motion, from a position 2 ft below its equilibrium position, by giving it a downward velocity of 2 ft/sec. At the end of π seconds,

   (a) Determine whether the mass is above or below the equilibrium position and if so, by what distance.

   $X(\pi) = -2e^{-\pi}$

   above equilibrium by 0.09 ft

   (b) Determine the maximum mass that results in overdamped motion.

   $m < \frac{1}{4}$ slugs

4. A force of 400 Newtons stretches a spring 2 meters. A mass of 50 kilograms is attached to the end of the spring and is initially released from the equilibrium position with an upward velocity of 10 m/s. Where is the spring after 10 seconds? How long will it take the spring to come back to equilibrium?

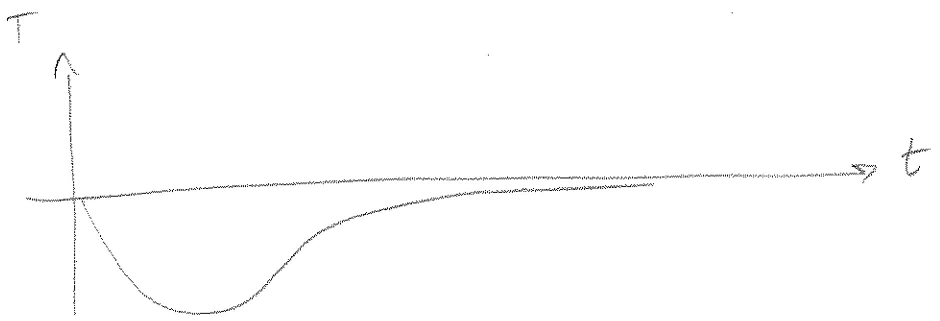
   $X(10) = -4.56473$ m

   $t = 1$ sec

5. A 10kg mass is attached to a spring with spring constant of 140N/m. The mass is suspended in a viscous medium with a damping coefficient of 90 N-sec/m. The mass is started in motion from the equilibrium position with an initial velocity of 1m/sec in the upward direction. Formulate and solve an initial value problem that models the given system; graph and interpret the results.

$10x'' + 90x' + 140x = 0 \quad , \; x(0) = 0 \; , \; x'(0) = 1$

$x(t) = -\frac{1}{5}e^{-2t} + \frac{1}{5}e^{-7t}$



over damped