Mechanics Based Problems

1. Using Mathematica, draw a direction field for the DE $\frac{dy}{dx} = x^2 - y^2$ then print out the file and draw solution curves for the following conditions: y(-2)=1, y(3)=0, y(0)=2, y(0)=0.

2. Using Mathematica, draw a direction field for the DE $\frac{dy}{dx} = \sin(x) * \cos(y)$ then print out the file and draw solution curves for the following conditions: y(0)=1, y(1)=0, y(3)=3, y(0)=-5/2. Print out the file.

Problem Solving Problems *For the following problems, estimate the proportionality constant, create a slope field and sketch a possible solution curve to answer the questions.*

1. Suppose you have just poured a cup of freshly brewed coffee with temperature 95°C in a room where the temperature is 20°C.

   (a) When do you think the coffee cools most quickly? What happens to the rate of cooling as time goes by? Explain.

a) cools quickly upfront

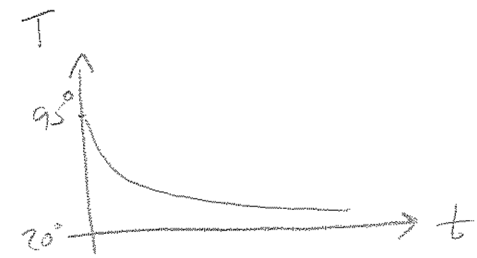
b) cooling decreases

PROVIDE YOUR EXPLANATION

   (b) **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. Write a differential equation that expresses Newton's Law of Cooling for this particular situation. What is the initial condition? In view of your answer to part (a), do you think this differential equation is an appropriate model for cooling?

$$\frac{dT}{dt} = k(T - 20^\circ)$$

$$T(0) = 95^\circ$$



   (c) Make a rough sketch of the graph of the solution of the initial-value problem (IVP) in part (b).

2. A thermometer is taken from an inside room to the outside, where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes it reads 30° F. What is the initial temperature of the inside room? Assume Newton's Law of Cooling applies. Recall: **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large.

$\bar{k} = -.1875$

Using a DE Plot:

$65^\circ \leq T(0) \leq 70^\circ$

3. A small metal bar, whose initial temperature was 20° C, is dropped into a large container of boiling water. How long will it take the bar to reach 90° C if it is known that its temperature increases 2° C in the 1st second? How long will it take the bar to reach 98° C? Assume Newton's Law of Cooling applies. Recall: **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large.

$T(t) = 90^\circ \Rightarrow \overbrace{t \approx 100\,s}^{\text{Read off DE Plot}}$

$T(t) = 98^\circ \Rightarrow t \approx 145\,s$

4. The population of a community is said to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take the population to triple? To quadruple?

$\bar{k} = 0.15$

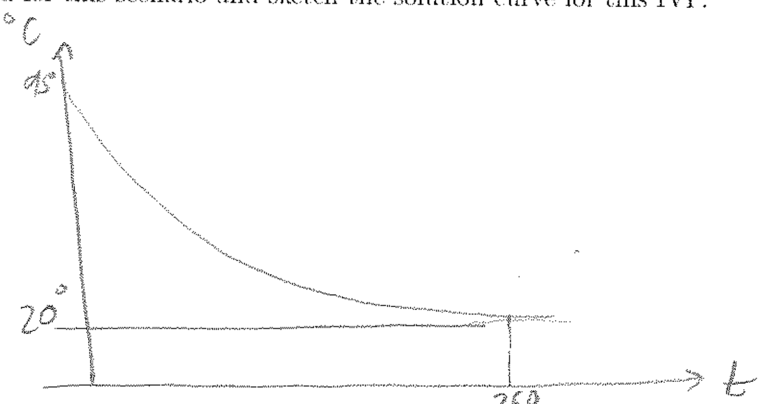
DE Plot $\Rightarrow$ triples $\rightarrow$ 7.5 years
quadruple $\rightarrow$ 9 years

5. A freshly brewed cup of coffee, with a temperature 95°C, is poured into a cup in a room with an ambient temperature of 20°C. Suppose it is known that the coffee in that cup cools at a rate of 1°C per minute when its temperature 70°C.

   (a) What is the initial value problem modeled by this situation?

$$\frac{dT}{dt} = -\frac{1}{50}(T - 20), \quad T(0) = 95°C$$

   (b) Create a slope field for this scenario and sketch the solution curve for this IVP.



   (c) What is the limiting value of the coffee's temperature? About how long will it be before it reaches that value?

limiting value = 20°

$t \approx 260s$

4. The population of a community is said to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take the population to triple? To quadruple?

Let $P_0 = 1$ So $P(0) = 1$
$P(5) = 2$

$$\frac{dP}{dt} = KP$$

K - proportionality constant

P - Population at time t

$P(0) = P_0$

$P(5) = 2P_0$

Find $\frac{dP}{dt} = \frac{\Delta P}{\Delta t} = \frac{1}{5} = .2$

Find K

$.2 = K$ $K = .2$ $\overline{K} = .15$

$.2 = 2K$ $K = .1$

DEPLOT $\approx$ 7.5 years Triples, 9 years Quadruples

5. A freshly brewed cup of coffee, with a temperature 95°C, is poured into a cup in a room with an ambient temperature of 20°C. Suppose it is known that the coffee in that cup cools at a rate of 1°C per minute when its temperature 70°C.

(a) What is the initial value problem modeled by this situation?

$$\frac{dT}{dt} = K(T - T_s)$$

T - Temp. coffee after time t
K - proportionality constant
Ts - Surrounding Temp

$T(0) = 95°C$

$1°C = K(70 - 20)$

$K = -1/50$

$$\frac{dT}{dt} = -\frac{1}{50}(T - 20)$$ ANS

(b) Create a slope field for this scenario and sketch the solution curve for this IVP.

See Code attached

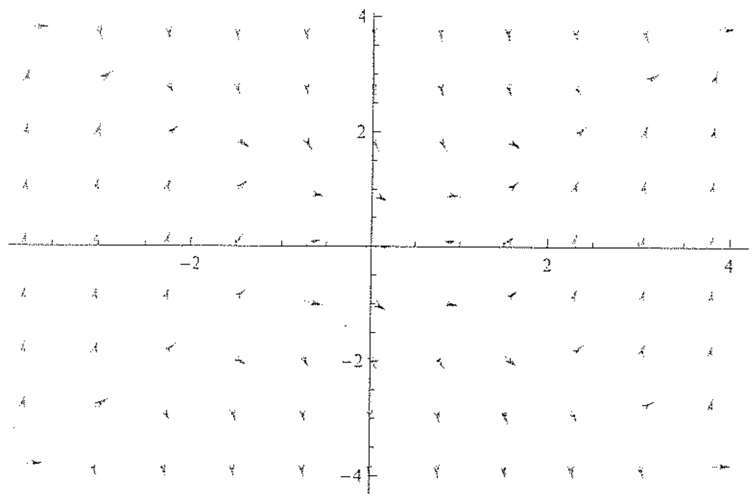
(c) What is the limiting value of the coffee's temperature? About how long will it be before it reaches that value?

limiting value = 20°, which it will reach around $t = 260$ minutes

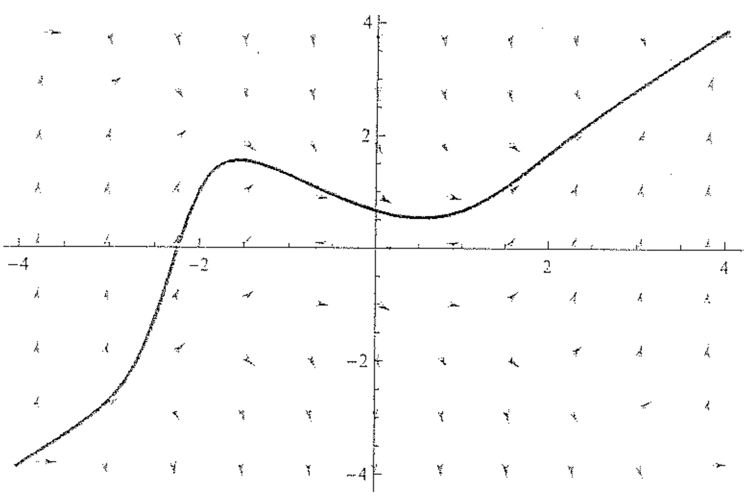
In[6]:= << DiffEqs`DEGraphics`

Mechanics Based Problem #1

In[15]:= DEPlot[x^2 - y^2, {x, -4, 4}, {y, -4, 4}, InitialPoints → None]

Out[15]=


In[20]:= DEPlot[x^2 - y^2, {x, -4, 4}, {y, -4, 4}, InitialPoints → {{-2, 1}}]

Out[20]=


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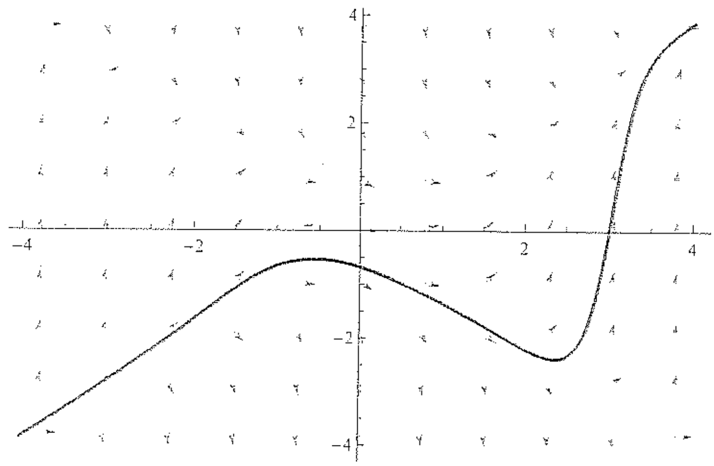
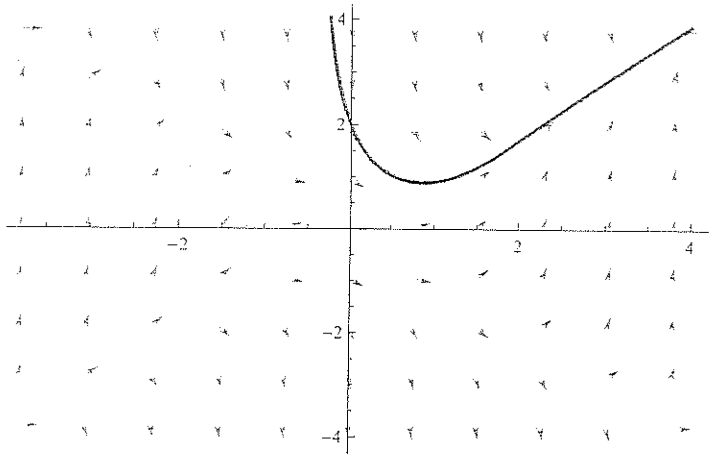
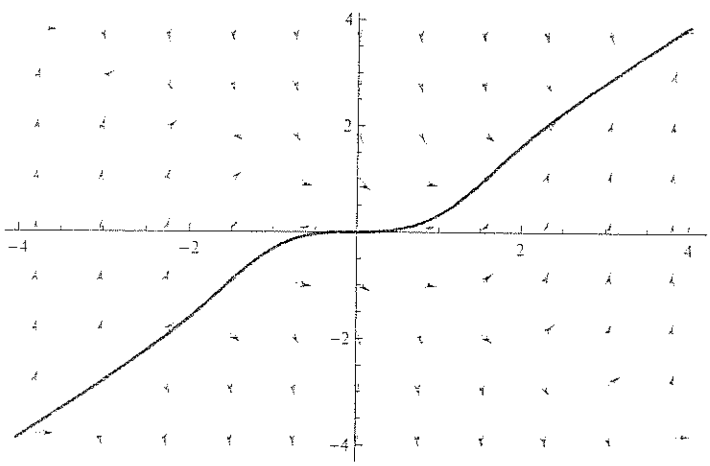
DEPlot[x^2 - y^2, {x, -4, 4}, {y, -4, 4}, InitialPoints → {{3, 0}}]
```




```
DEPlot[x^2 - y^2, {x, -4, 4}, {y, -4, 4}, InitialPoints → {{0, 2}}]
```




```
DEPlot[x^2 - y^2, {x, -4, 4}, {y, -4, 4}, InitialPoints → {{0, 0}}]
```



Mechanics Based Problem #2

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DEPlot[Sin[x]*Cos[y], {x, -4, 4}, {y, -4, 4}, InitialPoints → None]
```
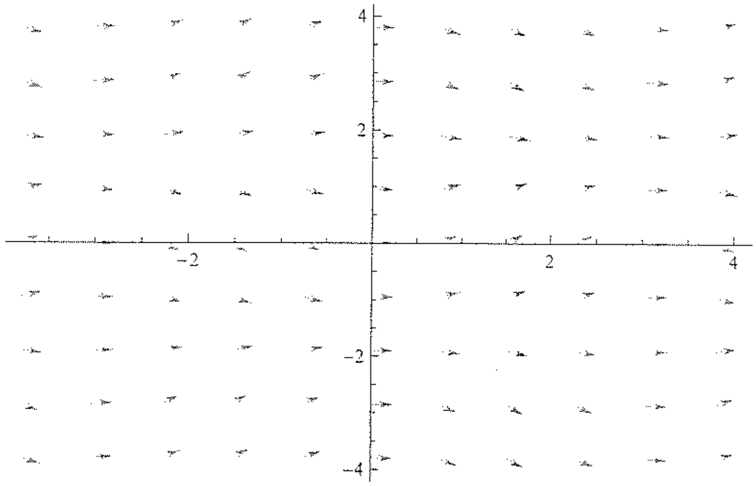
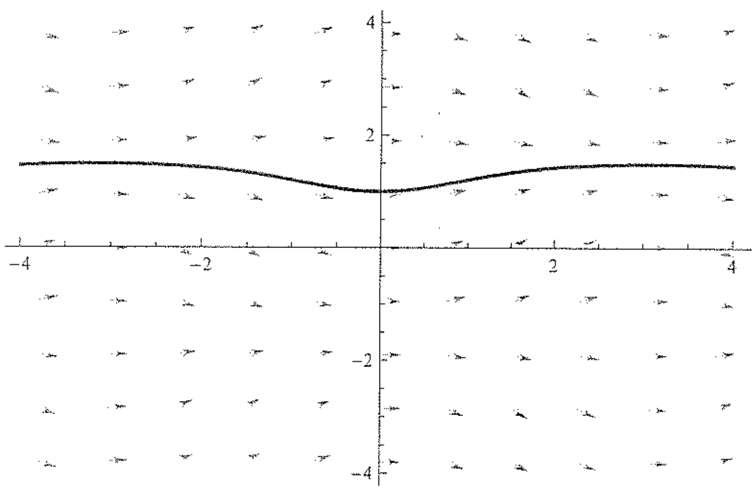
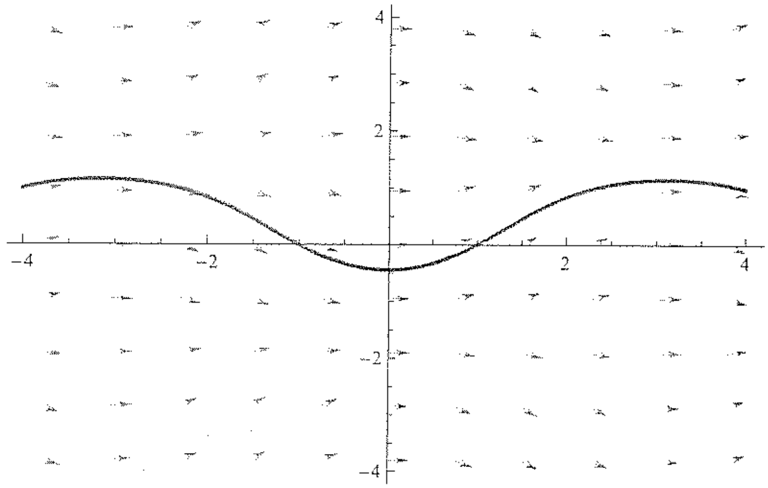



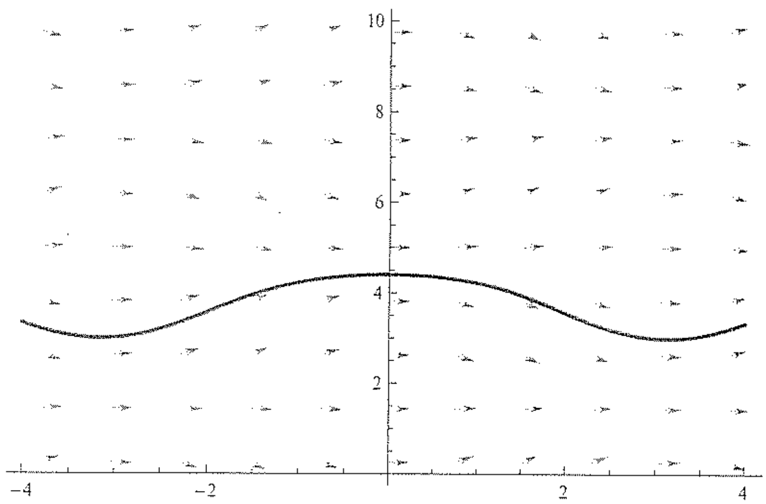

```
DEPlot[Sin[x]*Cos[y], {x, -4, 4}, {y, -4, 4}, InitialPoints → {{0, 1}}]
```



```
DEPlot[Sin[x] * Cos[y], {x, -4, 4}, {y, -4, 4}, InitialPoints → {{1, 0}}]
```




```
DEPlot[Sin[x] * Cos[y], {x, -4, 4}, {y, -0, 10}, InitialPoints → {{3, 3}}]
```




```
DEPlot[Sin[x] * Cos[y], {x, -4, 4}, {y, -8, 0}, InitialPoints → {{0, -5 / 2}}]
```