Lesson # 19 Answers

Lesson 19 - Vector Functions II

Mechanics Based Problems

1. Find the position vector of a particle that has the given acceleration and the specified initial velocity and position. Then use Mathematica to graph the path of the particle.

   (a) $\mathbf{a}(t) = \mathbf{i} + 2\mathbf{j} + 2t\mathbf{k}$, $\mathbf{v}(0) = \mathbf{0}$, $\mathbf{r}(0) = \mathbf{i} + \mathbf{k}$

$\vec{r}(t) \left\langle \frac{t^3}{3} + 1, t^2, \frac{t^3}{3} + 1 \right\rangle$

Ans

   (b) $\mathbf{a}(t) = t\mathbf{i} + t^2\mathbf{j} + \cos 2t\mathbf{k}$, $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$, $\mathbf{r}(0) = \mathbf{j}$

$\vec{r}(t) = \left\langle \frac{t^3}{6} + t, \frac{t^4}{12} + 1, -\frac{\cos(2t)}{4} + t + \frac{1}{4} \right\rangle$

Problem Solving Problems

1. The position function of a particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum?

$t = 4$

Ans

2. What force is required so that a particle of mass m has the position function $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$?

$F = m\langle 6t, 2, 6t \rangle$

Ans

3. A force with magnitude 20 N acts directly upward from the xy-plane on an object with mass 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and its speed at time t.

1) $r(t) = \left\langle t, -t, \frac{5t^2}{2} \right\rangle$

2) Speed $= \sqrt{25t^2 + 2}$

AWS