MA205 - Integral Calculus
Lesson 42: Modeling with Differential Equations

Mechanics Based Problems

1. For the following problems state the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear, homogenous or non-homogenous.

   (a) $(1-x)y'' - 4xy' + 5y = \cos(x)$

   (b) $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$

   (c) $(\sin(\theta))y''' - (\cos(\theta))y' = 2$

2. For the following problems verify that the indicated function is a solution of the given differential equation.

   (a) $2y' + y = 0, \quad y = e^{-x/2}$

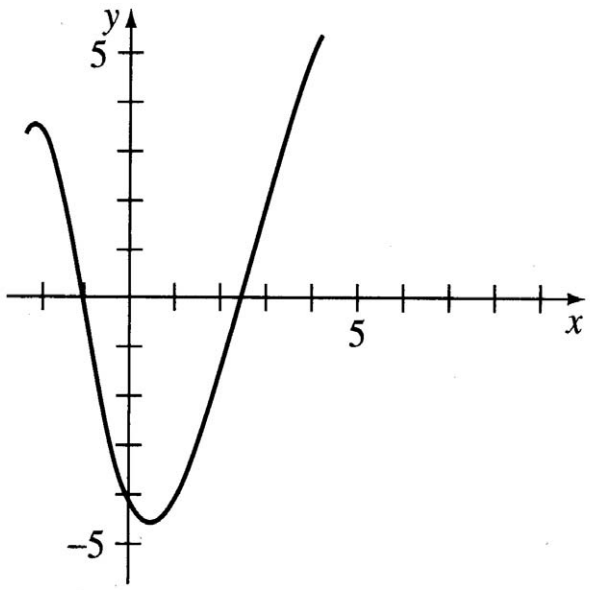
   (b) $y'' + y' - 6y = 0, \quad y_1 = c_1e^{2x}, \quad y_2 = c_1e^{2x} + c_2e^{-3x}$

(c) $xy' + y = 2x, \quad y = x - x^{-1}$

3. Given that $y = \frac{1}{x^2+c}$ is a solution of the first order DE $y' + 2xy^2 = 0$, find a solution of the first order IVP consisting of this differential equation and the initial condition $y(2) = 1/3$.

4. Given the following pairs of initial conditions and a graph of the particular solution to a second order differential equation of the form $d^2y/dx^2 = f(x, y, y')$ choose at least one set of initial conditions that agree with the solution curve.

   (a) $y(1) = 1$, $y'(1) = -2$
   (b) $y(-1) = 0$, $y'(-1) = -4$
   (c) $y(1) = 1$, $y'(1) = 2$
   (d) $y(0) = -1$, $y'(0) = 2$
   (e) $y(0) = -1$, $y'(0) = 0$
   (f) $y(0) = -4$, $y'(0) = -2$


y
5
5
x
−5

Problem Solving Problems

1. Find values of r so that the function $y = e^{rx}$ is a solution of the given differential equation. Explain your reasoning.

   (a) $y'' + y' - 6y = 0$

   (b) $y'' + 2y' + y = 0$

2. Find values of r so that the function $y = x^r$ is a solution of the differential equation,

$$xy'' + 2y' = 0$$

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3. (a) Show that every member of the family of functions $y = Ce^{x^2/2}$ is a solution of the differential equation $y' = xy$.

   (b) Illustrate part (a) by graphing several members of the family of solutions on a common screen.

   (c) Find a solution of the differential equation $y' = xy$ that satisfies the initial condition $y(0) = 5$.

   (d) Find a solution of the differential equation $y' = xy$ that satisfies the initial condition $y(1) = 2$.

4. A population of creatures is modeled by the differential equation

$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

(a) For what values of P is the population increasing?

(b) For what values of P is the population decreasing?

(c) What are the equilibrium solutions?

5. Psychologists interested in learning theory study **learning curves**. A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of time t. The derivative dP/dt represents the rate at which performance improves.

   (a) When do you think P increases most rapidly? What happens to dP/dt as t increases? Explain.

   (b) If M is the maximum level of performance of which the learner is capable, explain why the differential equation

   $$\frac{dP}{dt} = k(M - P), \qquad k > 0$$

   is a reasonable model for learning.

   (c) Make a rough sketch of a possible solution of this differential equation.