Lesson 45 - Euler's Method

Objectives

- Approximate the values of the solution to a first order differential equation using Euler's method.
- Understand how Euler's method uses the concept of slope in order to solve first order differential equations.
- Relate numerical solutions to graphical solutions of first order differential equations.
- Learn how to model a mixing problem with a differential equation.

READ

- Stewart, Chapter 9.2, pages 575-577
- Stewart, Chapter 9.3, pages 584-585 (through the middle of page 585; stop at "Solving this separable differential equation...")

THINK ABOUT

- What is required in order to use Euler's method?
- When do functions have places where there is no slope?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

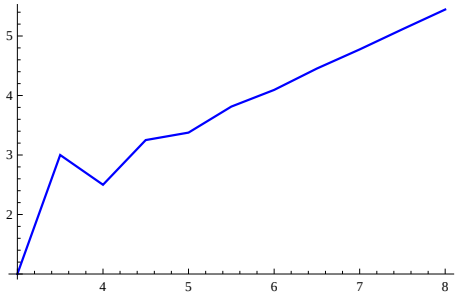
Here is an example of Euler's Method written in Mathematica Code:

```
slope[x_, y_] = 2*x - 3*y + 1; (*define the differential equation*)

step = .5; (*define step size*)
x[0] = 3; (*define initial x value*)
y[0] = 1; (*define initial y value*)
iterations = 10;
Do[
 x[i] = x[i - 1] + step;
 y[i] = y[i - 1] + step*slope[x[i - 1], y[i - 1]],
 {i, 1, iterations}]

(*Prints Euler's Method Solution Values*)
values = TableForm[Table[{x[i], y[i]}, {i, 0, iterations}], TableHeadings → {None, {"x", "y"}}]

(*Graphs Euler's Method Solution Values*)
eulersoln = ListPlot[Table[{x[i], y[i]}, {i, 0, iterations}], Joined → True, PlotStyle → {{RGBColor[0, 0, 1], Thickness[0.005]}}]
```

It will solve the problem $\frac{dy}{dx} = 2x - 3y + 1$ with initial condition $y(3) = 1$ (and other problems with appropriate modifications). Your result should be the table of values and plot of these values as shown below:

x	y
3	1
3.5	3.
4.	2.5
4.5	3.25
5.	3.375
5.5	3.8125
6.	4.09375
6.5	4.45313
7.	4.77344
7.5	5.11328
8.	5.44336