Block III Objectives and Calendar

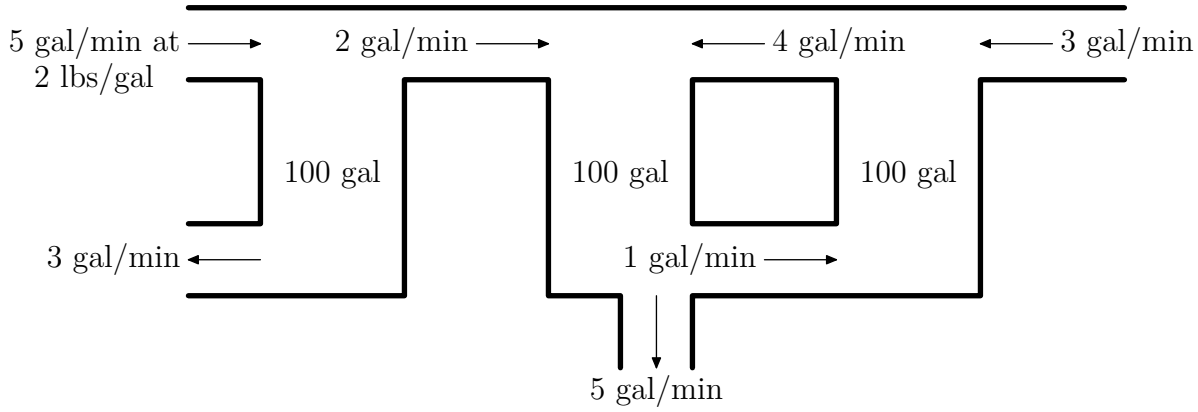
Block III Objectives

1. Understand the vocabulary of differential equations:
   (a) Independent vs. Dependent Variable.
   (b) Order of a Differential Equation.
   (c) Linearity of a Differential Equation.
   (d) Analytic Solution, Graphical Solution, Numerical Solution.
   (e) General vs. Particular Solutions.
   (f) Stable and Unstable Equilibrium.
2. Determine if a function is a solution to a given Differential Equation or Initial Value Problem.
3. Model the following scenarios with a first order differential equation:
   (a) Proportional Growth or Decay.
   (b) Newton's Law of Cooling.
   (c) Mixing Problems.
   (d) Logistical Growth.
4. Solve first order Differential Equations using the following techniques:
   (a) Separation of Variables (implies without technology).
   (b) Euler's Method.
   (c) Slope Fields.
5. Describe the long term behavior of the solution of a first order differential equation.
6. Model the following scenarios with a second order differential equation:
   (a) Damped and Undamped Harmonic Oscillators.
   (b) Forced Oscillation.
7. Solve second order Differential Equations using the following techniques and or concepts:
   (a) The characteristic equation or auxiliary equation.
   (b) The principle of superposition.
8. Describe the long term behavior of the solution of a second order, homogeneous differential equation modeling motion in a spring-mass system.
9. Solve a system of first order, homogeneous differential equations.

Block III Goal Problems

By the end of this block of instruction, you should be very comfortable analyzing problems similar to these:

1. Using the figure below derive three differential equations that specify the amount of dye in each of the three 100 gallon chambers. Assume all chambers start off with 0 lbs of dye.



2. Air containing 0.06% carbon dioxide is pumped into a room whose volume is 8000 cubic feet. The air is pumped in at a rate of 2000 cubic feet per minute, and the circulated air is then pumped out at the same rate. If there is an initial concentration of 0.2% carbon dioxide in the room, determine the subsequent amount in the room at time t. What is the concentration at 10 minutes? What is the equilibrium concentration of carbon dioxide?

3. The number $N(t)$ of supermarkets throughout the country that are using a computerized check out system is described by the initial value problem:

$$\frac{dN}{dt} = N(1 - 0.0005N), N(0) = 1$$

   (a) Use a slope field to predict how many supermarkets are expected to adopt the new procedure over a long period of time. Sketch by hand your solution curve on your slope field.

   (b) Solve the initial value problem and then graph your solution. Compare the two curves.

   (c) How many supermarkets are expected to adopt the new technology in ten years?

4. A 16 lb weight stretches a spring 4 ft. The spring-mass system is in a medium with a damping constant of 4.5 lb-sec/ft. What is the solution function describing the position of the mass at any time if the mass is released from 2 ft below the equilibrium position with an initial velocity of 4 ft/sec downward?

5. A mass of 1 slug is attached to a spring whose constant is 25/4 lb/ft. Initially the mass is released 1 ft above the equilibrium position with a downward velocity of 3 ft/sec, and the subsequent motion takes place in a medium that offers a damping force numerically equal to 3 times the instantaneous velocity. An external force given by $f(t) = 10$ (in pounds) is driving the system. Formulate and solve an initial value problem that models the given system. Interpret your results.