MA205 - Integral Calculus
Lesson 36: Polar Regions I

Mechanics Based Problems

1. Sketch the regions in the xy plane consisting of points whose polar coordinates satisfy the given conditions.

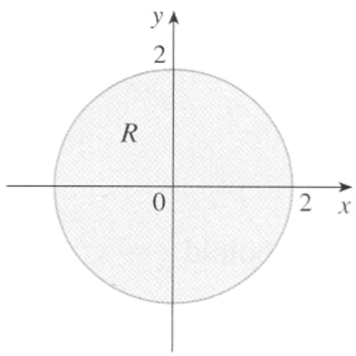
   (a) $r > 1$

   (b) $1 \leq r \leq 3$ and $-\pi/4 \leq \theta \leq \pi/4$

   (c) $2 < r < 3$ and $5\pi/3 \leq \theta \leq 7\pi/3$

2. A region R is shown below. Describe the regions by establishing the ranges for both r and θ if the region is better described by polar coordinates and x and y if the region is better described by cartesian coordinates. Then, establish an iterated integral over the region depicted using an arbitrary function $f(x, y)$. For example region 1 is a solid circle described as $0 \leq r \leq 2$ and $0 \leq \theta \leq 2\pi$ and $\int_0^{2\pi} \int_0^2 f(r\cos\theta, r\sin\theta) r dr d\theta$ would be the iterated integral of choice.

1.



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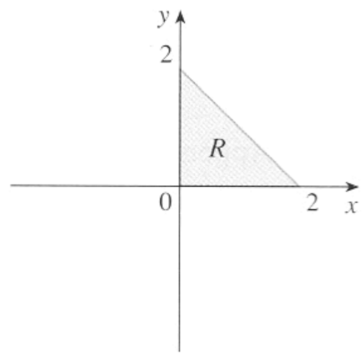

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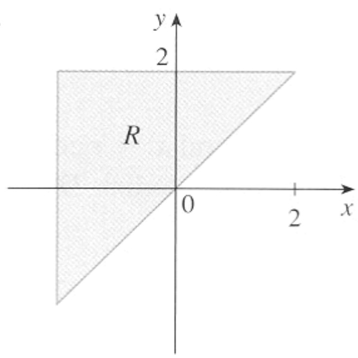

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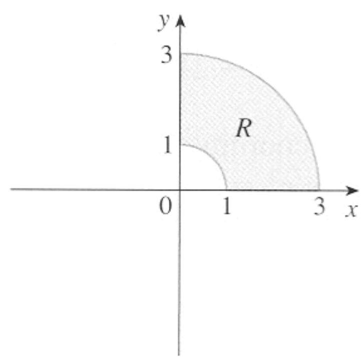

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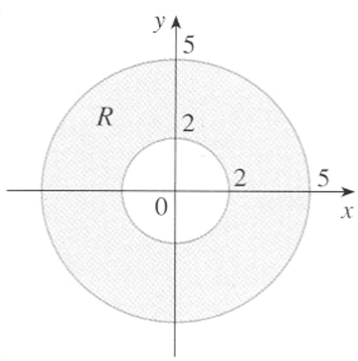

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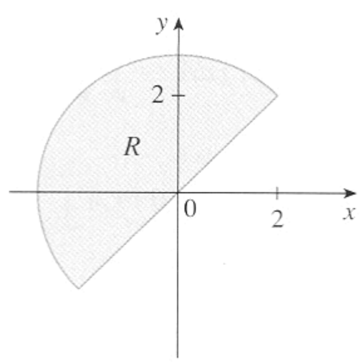

3. Sketch the region whose area is given by the integral and evaluate the integral:

$$\int_{\pi}^{2\pi} \int_{4}^{7} r dr d\theta$$

4. Evaluate the given integrals by changing to polar coordinates:

$$\int \int_{R} y e^{x} dA$$

where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 25$

$\boxed{\text{Problem Solving Problems}}$

1. Use polar coordinates to find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$

2. A swimming pool is circular with a $40ft$ diameter. The depth is constant along east-west lines and increases linearly from $2ft$ at the south end to $7ft$ at the north end. Find the volume of water in the pool.

3. A small yard sprinkler distributes water in a circular pattern of radius 15ft. It supplies water to a depth of $e^{-\sqrt{r}}$ feet per hour at a distance of r feet from the sprinkler.

   (a) What is the total amount of water supplied per hour to the region inside the circle of radius R ft centered at the sprinkler?

   (b) Determine an expression for the average amount of water per hour per square foot supplied to the region inside the circle of radius R ft.

   (c) Compute the average amount of water per hour per square foot supplied to the yard, based on the maximum range of the sprinkler.

4. A lamina occupies the region inside the circle $x^2 + y^2 = 4$ but outside the circle $x^2 + y^2 = 1$ in the first quadrant. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

5. The town of East Podunk is shaped like a series of two concentric circles. See the diagram provided. The population density in the inner city (a radius of 5 miles measured from the center of town) is determined to be $\rho(x, y) = 1200e^{-(x^2+y^2)}$. The population density in the outer region (out to a radius of 7.5 miles) has a constant population density of 50 people per square mile. What is the *total* population of East Podunk? What is the *average* population density of the town?