

**MA205 - Integral Calculus**  
**Lesson 10: The Substitution Rule I**

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Mechanics Based Problems

1. Evaluate the following integrals by making the given substitutions:

(a)  $\int x(4 + x^2)^{10} dx, \quad u = 4 + x^2$

(b)  $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx, \quad u = \sqrt{x}$

2. Evaluate the indefinite integrals:

(a)  $\int ye^{y^2} dy$

(b)  $\int t^2 \cos(1 - t^3) dt$

3. Evaluate the definite integrals:

(a)  $\int_0^{\pi/2} e^{\sin(x)} \cos(x) dx$

(b)  $\int_0^{\pi/4} \sin(4t) dt$

Problem Solving Problems

1. A bacteria population starts with 400 bacteria and grows at a rate  $r(t) = (450.268t)e^{1.12567t^2}$  bacteria per hour. How many bacteria will there be after three hours?

2. Alabama Instruments Company has set up a production line to manufacture a new calculator. The rate of production of these calculators after  $t$  weeks is:

$$\frac{dx}{dt} = 5000 \left( 1 - \frac{100}{(t+10)^2} \right) \text{ calculators/week}$$

(Notice that production approaches 5000 per week as time goes on, but the initial production is lower because of the workers' unfamiliarity with the new techniques.) Find the number of calculators produced from the beginning of the third week to the end of the fourth week.