Lesson 4 - Refining Estimates

Objectives

- Understand the area between a curve and the x axis is the limit of the sum of areas of rectangles or trapezoids.
- Improve all estimations by increasing the number of subintervals.

READ

- Review previous readings on approximating
- Stewart Section 7.7 pages 495-498

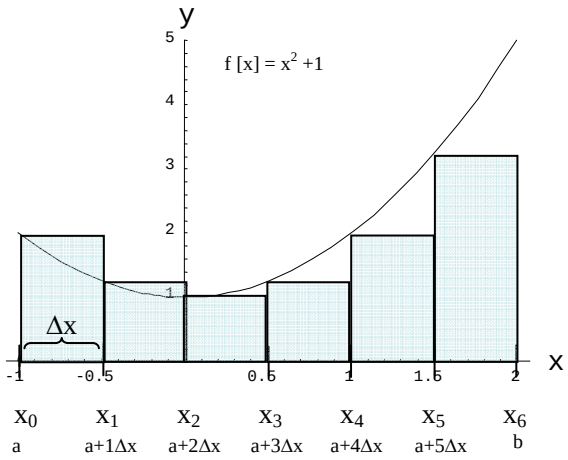
THINK ABOUT

- What is an appropriate subinterval for a given problem?
- Should you just jump to a subinterval of 0.00000000000001 right away?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

The following graphs and commands depict all the methods of approximating area between a curve and the x axis you have learned. For the code below, when `a=-1`, `b=2`, and `f[x_]=$x^2$+1`,

Left endpoint approximation



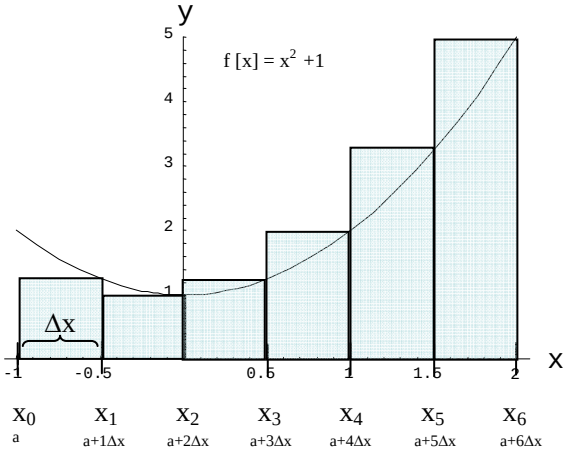
- **Text version of formula**

  $L_n = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x$, where $\Delta x = \frac{b-a}{n}$

***Mathematica* version of formula**

```
deltaX = (b - a)/n;
x[i_] = a + i * deltaX;
L[n_] = Sum[f[x[i - 1]] * deltaX, {i, 1, n}]
```

Right endpoint approximation



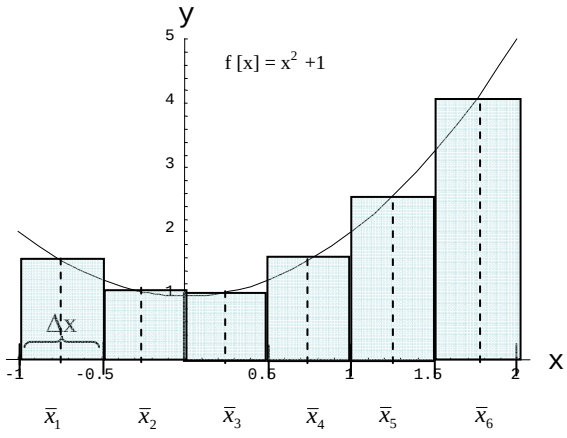
- **Text version of formula**

  $R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$, where $\Delta x = \frac{b-a}{n}$

***Mathematica* version of formula**

```
deltaX = (b - a)/n;
x[i_] = a + i * deltaX;
R[n_] = Sum[f[x[i]] * deltaX, {i, 1, n}]
```

Midpoint approximation



- **Text version of formula**

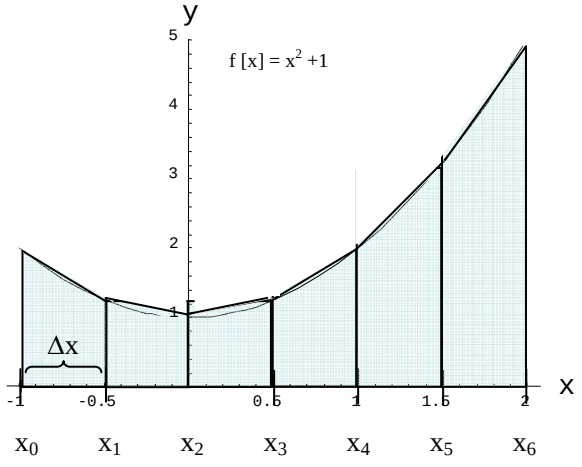
$M_n = \sum_{i=1}^{n} f(\overline{x}_i)\,\Delta x$, **where** $\Delta x = \frac{b-a}{n}$

$\overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

***Mathematica* version of formula**

```
deltaX = (b - a)/n;
x[i_] = a + i * deltaX;
M[n_] = Sum[f[(x[i - 1] + x[i])/2], {i, 1, n}] * deltaX
```

Trapezoidal approximation



- **Text version of formula**

$T_n = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{f[x[i-1]]\,\Delta x + f[x[i]]\,\Delta x}{2}\right]$, **where** $\Delta x = \frac{b-a}{n}$

***Mathematica* version of formula**

```
deltaX = (b - a)/n;
x[i_] = a + i * deltaX;
T[n_] = Sum[(f[x[i - 1]] + f[x[i]])/2, {i, 1, n}] * deltaX
```