Lesson 2 - Estimations From Functions and Data Sets I

Objectives

- Choose sample points that lead to upper and lower estimates.
- Estimate the area of a general region using left end points and right end points.
- Estimate total distance traveled.
- Estimate the area between a curve and the x axis numerically when given a function using left end points, right end points, midpoints, and trapezoids.

READ

- Stewart Section 5.1 pages 354-363
- Additional Course Notes, Student Guide, Lesson 2

THINK ABOUT

- What is a sample point?
- What does it mean to be an upper estimate or a lower estimate?
- What does it mean to be an upper bound or a lower bound?
- How would you describe the least upper bound or greatest lower bound?
- Why do some sample points lead to upper estimates and some lead to lower estimates?
- What happens when your data drops below the x-axis?

MATHEMATICA COMMANDS AND TASKS YOU NEED TO KNOW

Entering data into mathematica is done by creating a list. If you wanted to put the data points $(0, 1), (2, 4), (4, 6), (6, 8)$ into mathematica you would type:

```
data={{0,1},{2,4},{4,6},{6,8}}
```

Then to plot this data you would use the command:

```
ListPlot[data]
```

You can connect the dots if you would like to with the command `PlotJoined→ True`. You should always label your axes with the command `AxesLabel→{"x axis label name","y axis label name"}`

Additional Course Notes

There is a difference between the Midpoint rule and the Trapezoidal rule.

- The Midpoint rule (figure 1.c) uses approximating rectangles whose heights are obtained by evaluating $f(x)$ at the midpoint of the x interval.

$$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

  where $\Delta x = \frac{b-a}{n}$, $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$, and $x_i = a + i\Delta x$.

- The Trapezoidal rule (figure 1.d) uses approximating trapezoids under the curve. In fact, the Trapezoidal rule is the average of the Left and Right endpoint rules. (See figures 1.a and 1.b)

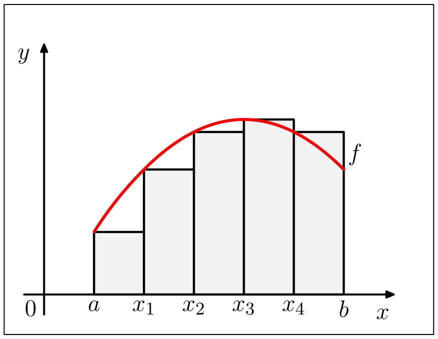
$$T_n = \frac{1}{2}\left[\sum_{i=1}^{n} f(x_{i-1})\Delta x + \sum_{i=1}^{n} f(x_i)\Delta x\right] = \frac{\Delta x}{2}\left[\sum_{i=1}^{n} f(x_{i-1}) + \sum_{i=1}^{n} f(x_i)\right]$$

  Recall the area of a trapezoid whose sides are a, b, and c (figure 1.e) is $\frac{c}{2}(a+b)$. So the area of the ith trapezoid is

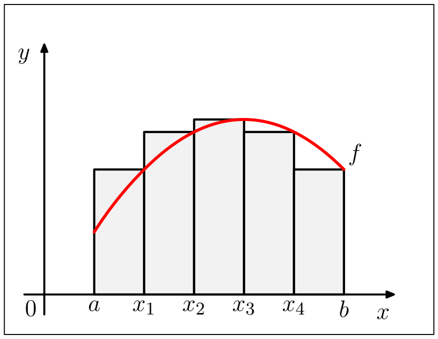
$$\frac{\Delta x}{2}\left[f(x_{i-1}) + f(x_i)\right]$$

  making the accumulation of all trapezoids

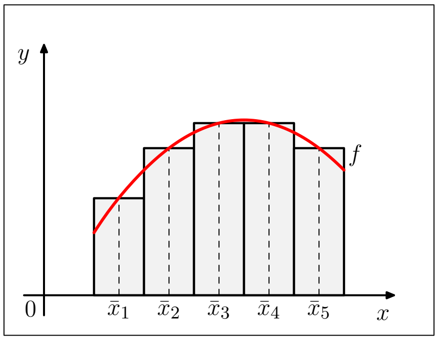
$$\sum_{i=1}^{n}\left[\frac{f(x_{i-1}) + f(x_i)}{2}\right]\Delta x$$



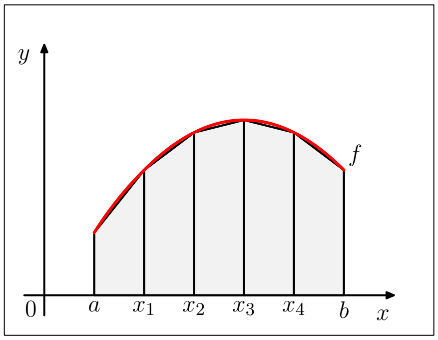
(a) Left Endpoint Approximation



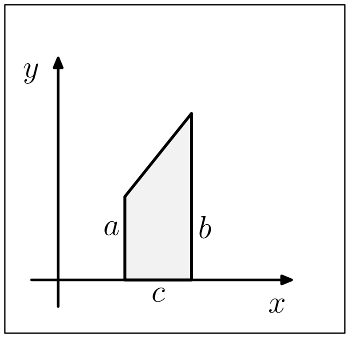
(b) Right Endpoint Approximation



(c) Midpoint Approximation



(d) Trapezoidal Approximation



(e) The Trapezoid

Figure 1: Approximate Integrations